These problems are intended to serve as supplementary practice to help get you thinking, in somewhat more precise way, about sets, proofs and functions.

1. If $\mathcal{U} = \{0, 2, 4, 6, 8, 10\}$, $A = \{0, 4, 8\}$, $B = \{2, 4, 6, 8\}$ and $C = \{0, 6, 10\}$ determine each of the following
   (a) A'.
   (b) $(A \cup B) \cap C'$
   (c) $A' \cap B' \cap C'$
   (d) $A - B'$
2. Show that, in general, $A - B \neq B - A$. If $A - B = B - A$, what can be said about A and B?
3. For arbitrary sets $F, G, \mathcal{U}$, place the following in numerically non-decreasing order: $|F|$, $|F \cup G|$, $|F \cap G|$, $|\emptyset|$, $|\mathcal{U}|$.
4. For each of the following, give a relation on the set $\{a, b, c, d\}$ that satisfies the conditions:
   (a) Reflexive and symmetric, but not transitive
   (b) Reflexive and transitive, but not symmetric
   (c) Symmetric and transitive, but not reflexive
5. Consider the successor relation S on $\mathbb{N}$ defined as follows:
$$S = \{(x, y) | y = x + 1\}$$
   What relation is the reflexive transitive closure of S?
6. Say which of the following functions are onto and which are one-to-one. Which, if any, are bijections?
   (a) *ceiling* : $\mathbb{R} \to \mathbb{Z}$ (where *ceiling* is the function which, for all real x, returns the least integer greater than or equal to x.)
   (b) $f : \mathbb{Z} \to \mathbb{Z}$, where $x \mapsto x^2$
   (c) $f : \mathbb{N} \to \mathbb{Z}$ defined as follows:
$$f(n) = \begin{cases} n/2, & \text{if } n \text{ is even;} \\ -(n-1)/2, & \text{if } n \text{ is odd.} \end{cases}$$
7. A function $f : X \to Y$ is said to have a left inverse $l : Y \to X$ iff $l \circ f$ is the identity function on X (i.e., for all $x \in X, l \circ f(x) = x$). Prove that
   (a) if f has a left inverse l, then l is onto.
   (b) if f is one-to-one, then it has a left inverse.