

050.371/671 — Formal Methods in Cognitive Science
Problem Set 1
Due March 15, 1999

Problem 1

Construct a Turing Machine which computes the function $f(n) = 3n$ for n given in unary.

Problem 2

Construct Turing Machines which decide each of the following languages:

1. $\{w\bar{c}w \mid w \in \{a, b\}^*\}$
2. $\{w \in \{0, 1\}^* \mid w \text{ contains an equal number of 0s and 1s}\}$

Problem 3

Let A be the language containing only the single string s , where

$$s = \begin{cases} 0, & \text{if God does not exist;} \\ 1, & \text{if God exists.} \end{cases}$$

Is A decidable? Why or why not? (Note that the answer doesn't depend on your religious convictions.)

Problem 4

For each of the following languages, state whether it is recursive, R.E. but not recursive, or neither, and provide a proof. (Take $L(M)$ to indicate the language *accepted* by M .)

1. $L_e = \{M \mid L(M) = \emptyset\}$
2. $L_{ne} = \{M \mid L(M) \neq \emptyset\}$
3. $L_a = \{M \mid M \text{ halts on all inputs}\}$