

Formal methods, Fall 2008
Problem set 3 – infinite sets and proofs
Assigned: 9/18, Due: 9/25

[A] Show, by a direct (i.e. not contradiction, not induction) proof, that the union of any two countably infinite sets X and Y is countably infinite. (By “direct” I mean show that there exists a bijection, given X and Y , from some known countable set to $X \cup Y$, or vice versa. Your best strategy may be to start by considering particular examples, such as even and odd numbers. You may also find it useful to work with bijections from X and Y to disjoint subsets of \mathcal{N} at an intermediate stage in the proof, since you can’t directly assume X and Y don’t overlap themselves.)

[B] Show, by induction, that the union of any finite set of countably infinite sets is countably infinite. (Your result from [A] will be the cornerstone of this proof. Actually, even if you have trouble with [A], you can construct this proof by assuming the result there.)

[C] Show, using a proof by contradiction (and basic assumptions about intersection and/or universes), that $X \cap U = X$.