

Formal methods, Fall 2008
Problem set 1 – set theory
Assigned: 9/4, Due: 9/11

A | Some exercises

Do the following PMW chapter 1 exercises: 1(1-r), 3, 7(h-n), 9, 12. Re 12, does asking whether ‘autological’ is autological lead to a contradiction?

B | A semantic application

Background

Set theory pervades semantics. In dynamic semantics, a context is often taken to be a set of possible worlds. Each world represents a way that reality could be, as far as the discourse participants have mutually publically agreed. An “update” operation $+$ is a function whose domain is pairs of contexts and sentences to contexts; it returns contexts that have been updated with the information in the sentence. Intuitively, c is the context before an update, and $c + A$ for some sentence A gives you the context resulting from uttering A . If a sentence denotes a proposition, that is, a set of possible worlds (worlds where the sentence is true), $+$ is typically defined as follows for a non-complex sentence:

$$(1) \quad c + A \stackrel{\text{def}}{=} c \cap \llbracket A \rrbracket$$

If this is the only update operation, context updates are “monotonic” – we only get closer to knowing what possible world we are in. This is because it is defined in terms of \cap . Each update throws out the worlds where A is false.

Foreground

Sometimes update functions are defined for complex sentences in terms of updates of the pieces of the sentence. The following definition (from Heim 1983) provides a dynamic meaning for a conditional sentence.

$$(2) \quad \text{For any context } c, \text{ clause } A, \text{ and clause } B:$$
$$c + \text{If } A, B \stackrel{\text{def}}{=} c - ((c + A) - (c + A + B)) \quad (= c')$$

The variables c , A , and B here are all sets of possible worlds. Draw a Venn diagram showing the operation in terms of c , A , and B , marking the output context (call it c' for short). It may be helpful to pick particular clauses for A and B . Grads only: can you rewrite the definition so that c' is characterized using only the operations set intersection and union? If not, why not? (For the really ambitious, provide a proof.)

C | Spot the error

Optimality Theory involves competition between potential candidates for the output form (i.e. what a speaker pronounces, in the case of phonology). Given an input, the grammar considers the candidate outputs and considers what would be the optimal or most harmonic output. Consider the following (flawed) description of OT’s Gen (mangled from Tesar and Smolensky 1996):

- (3) *Gen*: Universal Grammar provides a function *Gen* whose domain is the set of inputs *I*, and whose range is the set of potential candidate structural descriptions for some input. Given an input, *Gen* maps from some input *I* to the candidates that are to be compared.

This is not formally coherent; what is the problem?

Bibliography

- Heim, Irene. 1983. On the projection problem for presuppositions. In *WCCFL 2: Second Annual West Coast Conference on Formal Linguistics*, ed. M. Barlow, D. Flickinger, and M. Wescoat, 114–125. Stanford, CA: CSLI.
- Tesar, Bruce, and Paul Smolensky. 1996. Learnability in optimality theory (short version). Technical report, Johns Hopkins University.