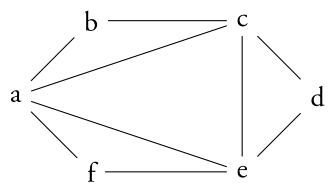
Formal methods, Fall 2008
Midterm
Assigned: 10/16, Due: 10/23

1 Graphs

[A] List all the automorphisms for the following undirected graph:

(1)



[B] Function composition (standardly written $\circ$, even outside of algebraic uses) is a way of putting together functions f and g such that the range of f is the domain of g. (i.e. $f : A \rightarrow B$ and $g : B \rightarrow C$, for some A, B, C; the three sets need not be distinct.) The composite function $g \circ f$ is defined as follows: $[g \circ f](x) = g(f(x))$ for every $x \in A$. (The domain of $g \circ f$ is A, and the range is C.)

Show that the set of automorphisms from part [a] forms a group with respect to function composition as the operation. (rhetorical question: does this look familiar? Think back to our examination of the ways of manipulating a square...)

[C] You might imagine that the following result would hold, given the previous section: two graphs will be (graph) isomorphic iff the sets of automorphisms of those graphs under the operation of function composition are (group) isomorphic. It does not. Give a counterexample. Is there some other relationship besides isomorphism between the graphs in the kind of circumstance involved in your counterexample (note, the answer may be different depending on what kind of counterexample you come up with)? What exactly is the information contained in a group of automorphisms?

(extra credit for all) Can you characterize the class or classes of graphs where this result would hold?

2 Generators

A group g is **generated** by a set of elements if the elements of G consist of all possible products of the generating elements and their inverses, with repetition allowed. (That is, it is the transitive closure of the operation with respect to these elements.) If a group is generated by a single element a, it is called a **cyclic group** and can be notated $\langle a \rangle$.

[A] Show that $\mathcal{Z}_6$ ($\langle\{0,1,2,3,4,5\}, \text{addition mod } 6\rangle$) is a cyclic group, and give its generator.

[B] List all cyclic subgroups of $\mathcal{Z}_6$.

[C] Is the group from 1b a cyclic group? List all of its cyclic subgroups.

[D] (Grads only) Show that for a group G that is generated by two elements a and b, if $ab = ba$, then G is abelian.

3 Trees and movement

You may have encountered constructions such as "wh-movement", where (metaphorically) some part of the tree moves to another part. The empty category t marks the source of the movement (in this example, the object of the preposition "to"):

(2) Alfonso talked to Bill.

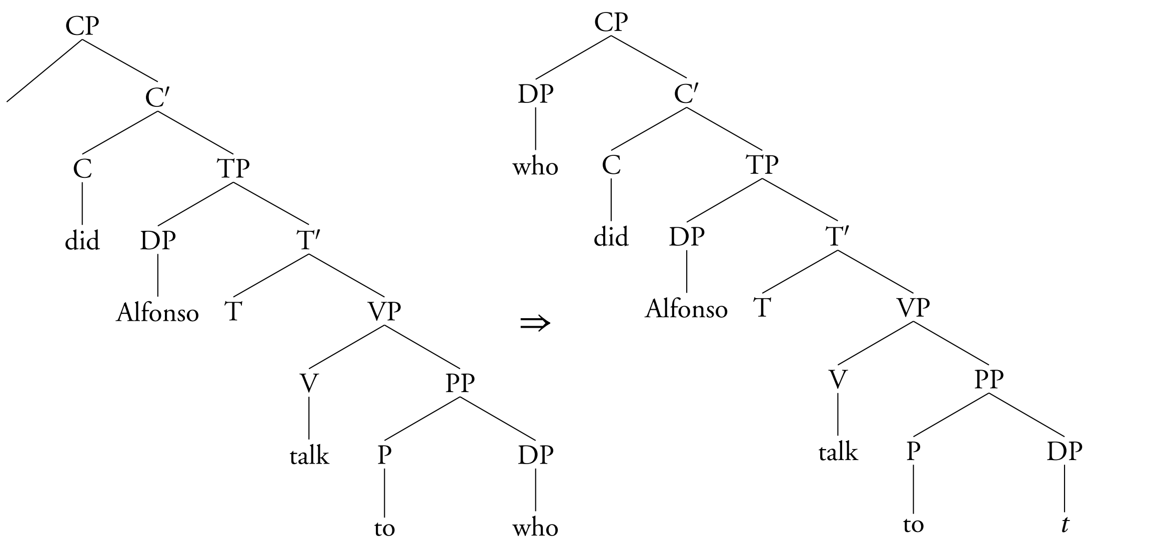
(3) Who did Alfonso talk to *t*?

You might wonder how this would fit in to our theory of linguistic trees. On one (very standard) view, the relationship between (3) and and the tree corresponding to (4) is *derivational* – a movement rule takes as input one true, and produces a new tree as its output.

(4) Did Alfonso talk to who?

Here is the movement rule illustrated with this example (Setting aside the position of “did”):

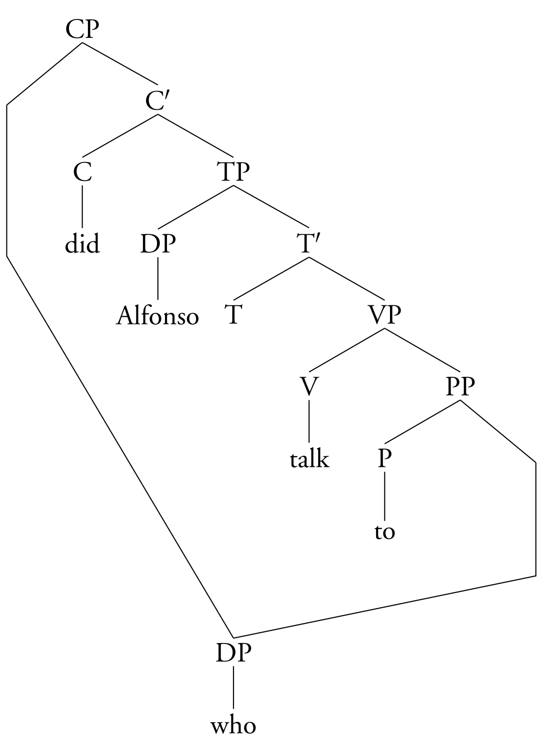
(5)



[**a**] Does this theory cause any problems for our theory of trees? Explain, making any new assumptions explicit.

[**b**] Several authors have recently proposed a different interpretation of movement. Instead of movement leaving behind a trace, it involves multi-attachment of the moved node. (Then, some rule tells you to pronounce the constituent in the highest position it is attached to.) On this view, the result of the movement rule would be the following structure:

(6)



Does this conception of movement cause any problems for our theory of trees? How could we revise our theory of trees (including any new assumptions you might have introduced in part [b]) to license structures of this kind? (Note that you do not need to deal with any constraints on movement itself here; assume that they are taken care of by independent principles.)

4 More worlds than there are?

(Obligatory for grads, optional for undergrads.)

The following is an argument using basic set theory (due to David Kaplan) that a possible world semantics for belief is incoherent (see Lewis 1986 §2.3). The goal of this problem is for you to evaluate the structure and any weaknesses of the argument.

Background: Suppose W is the set of possible worlds. A possible world is a complete specification of everything there is to know about one way the universe could be. A "proposition" is a subset of W, representing some interesting way of dividing up possible worlds. For instance, there is a proposition corresponding to the English sentence "the moon is made of cheese", containing only worlds where the moon is made of cheese. One traditional analysis of verbs like "believe" is that they take propositions as their arguments. So "Alfonso believes that the moon is made of cheese" would be true just in case Alfonso takes the actual world to be one of the worlds that is a member of the proposition corresponding to "the moon is made of cheese." Importantly, a belief report itself corresponds to a proposition, one that is true in exactly those worlds where Alfonso takes the actual world to be one of the worlds where the moon is made of cheese.

Here is the argument:

(7)
a. Suppose that the cardinality of $\mathscr{W}$ is K.
b. Each subset of this set is a proposition.
c. The set of subsets of $\mathscr{W}$ is $P(\mathscr{W})$, which we know by Cantor's theorem has a strictly greater cardinality than $\mathscr{W}$, i.e. 2^K.
d. Consider some person and time. For each proposition, it is possible that the person should have been thinking a thought at that time whose content would be specifiable by a sentence expressing that proposition; and that this should have been the person's only thought at that time.
e. So, there is a distinct possible situation corresponding to each such proposition (i.e. a way of individuating possible worlds).
f. Therefore, there must be 2^K possible worlds, contradiction the assumption in (a).

Note that Cantor's theorem importantly applies not just to finite cardinalities, but to non-finite cardinalities, which is probably what we are dealing with here. How could you respond to such an argument in defense of a possible world analysis of beliefs?

Bibliography

Lewis, David. 1986. *On the plurality of worlds*. Blackwell.